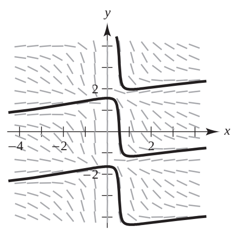
CHAPTER 15

Section 15.1 *(page 1098)*

1. $x^2 - 3xy + y^2 = C$ **3.** $3xy^2 + 5x^2y^2 - 2y = C$

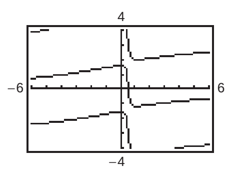
5. Not exact **7.** $\arctan \dfrac{x}{y} = C$ **9.** Not exact

11. (a) Answers will vary.



(b) $x^2 \tan y + 5x = \dfrac{11}{4}$

(c)



13. $y \ln(x - 1) + y^2 = 16$ **15.** $e^{3x} \sin 3y = 0$

17. Integrating factor: $\dfrac{1}{y^2}$

$\dfrac{x}{y} - 6y = C$

19. Integrating factor: $\dfrac{1}{x^2}$

$\dfrac{y}{x} + 5x = C$

21. Integrating factor: $\cos x$

$y \sin x + x \sin x + \cos x = C$

23. Integrating factor: $\dfrac{1}{y}$

$xy - \ln y = C$

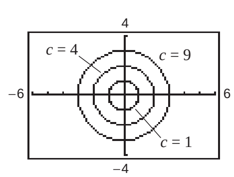
25. Integrating factor: $\dfrac{1}{\sqrt{y}}$

$x\sqrt{y} + \cos\sqrt{y} = C$

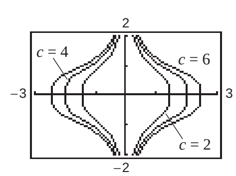
27. $x^4y^3 + x^2y^4 = C$

29. $\dfrac{y^2}{x} + \dfrac{x}{y^2} + C$ **31.** Proof

33. $x^2 + y^2 = C$



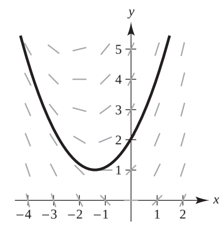
35. $2x^2y^4 + x^2 = C$



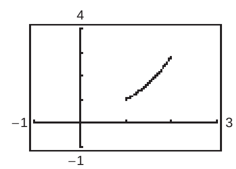
37. $x^2 - 2xy + 3y^2 = 3$ **39.** $C = \dfrac{5\left(x^2 + \sqrt{x^4 - 1{,}000{,}000x}\right)}{x}$

41.

Δx	0.50	0.25	0.10
Estimate	3.7798	3.9875	4.1207

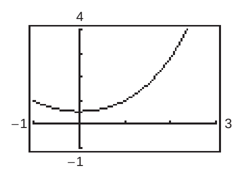


43. (a)

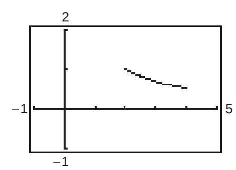


(b) $3y^{2/3} - x^2 = 2$

(c)

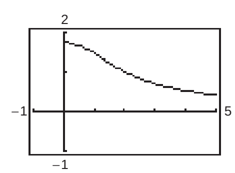


45. (a)

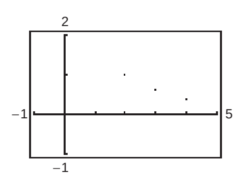


(b) $y^2(2x^2 + y^2) = 9$

(c)

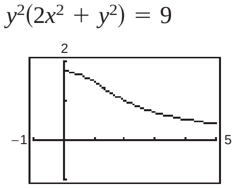


47. (a)



(b) $y^2(2x^2 + y^2) = 9$

(c)



Less accurate

49. False: $\dfrac{\partial M}{\partial y} = 2x, \quad \dfrac{\partial N}{\partial x} = -2x.$

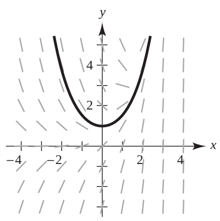
50. False: $ydx + xdy = 0$ is exact, but $xydx + x^2dy = 0$ is not exact.

51. True **52.** True

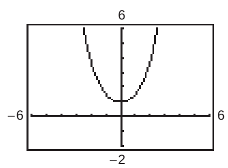
Section 15.2 *(page 1106)*

1. False: $y' + xy = x^2$ is linear. **2.** True

3. (a) Answers will vary. (b) $y = \frac{1}{2}(e^x + e^{-x})$



(c)



5. $y = x^2 + 2x + \dfrac{C}{x}$ **7.** $y = \frac{1}{2}(\sin x - \cos x) + Ce^x$

9. $y = -\frac{1}{13}(3 \sin 2x + 2 \cos 2x) + Ce^{3x}$

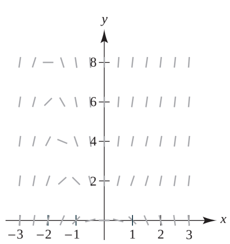
11. $y = \dfrac{x^3 - 3x + C}{3(x - 1)}$ **13.** $y = 1 + 4e^{-\tan x}$

15. $y = \sin x + (x + 1)\cos x$ **17.** $xy = 4$

19. $\dfrac{1}{y^2} = Ce^{2x^3} + \dfrac{1}{3}$ **21.** $y = \dfrac{1}{Cx - x^2}$

23. $y^{2/3} = Ce^{2x/3} - \dfrac{1}{4}(4x^3 + 18x^2 + 54x + 81)$

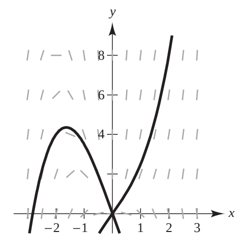
25. (a)



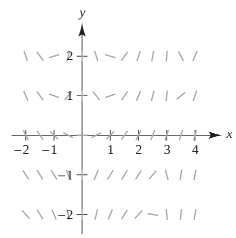
(b) $(-2, 4)$: $y = \dfrac{1}{2}x(x^2 - 8)$

$(2, 8)$: $y = \dfrac{1}{2}x(x^2 + 4)$

(c)



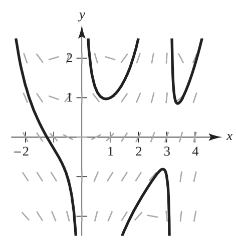
27. (a)



(b) $(1, 1)$: $y = \cos 1 \csc x - x \cot x + 1$

$(3, -1)$: $y = (3 \cos 3 - 2 \sin 3)\csc x - x \cot x + 1$

(c)



29. $I = \dfrac{E_0}{R} + Ce^{-Rt/L}$

31. $I = Ce^{-(R/L)t} + \dfrac{E_0}{R^2 + \omega^2L^2}(R \sin \omega t - \omega L \cos \omega t)$

33. $P = -\dfrac{N}{k} + \left(\dfrac{N}{k} + P_0\right)e^{kt}$

35. (a) \$583,098.01 (b) \$3,243,606.35

37. (a) $\dfrac{dQ}{dt} = q - kQ$

(b) $Q = \dfrac{q}{k} + \left(Q_0 - \dfrac{q}{k}\right)e^{-kt}$

(c) $\dfrac{q}{k}$

39. Proof

41. (a) $Q = 25e^{-t/20}$

(b) $-20 \ln\left(\dfrac{3}{5}\right) \approx 10.2$ minutes

(c) 0

43. (a) $t = 50$ minutes

(b) $100 - \dfrac{25}{\sqrt{2}} \approx 82.32$ pounds

45. c **46.** d **47.** a **48.** b **49.** $2e^x + e^{-2y} = C$

51. $x + xy^2 + \frac{1}{2}y^2 + 2y = C$

53. $y = Ce^{-\sin x} + 1$ **55.** $x^2y + \sin y = C$

57. $x^3y^2 + x^4y = C$ **59.** $y = \dfrac{e^x(x - 1) + C}{x^2}$

61. $x^4y^4 - 2x^2 = C$ **63.** $3 \arctan \dfrac{x}{y} - y = C$

Section 15.3 *(page 1115)*

1. Proof **3.** Proof **5.** $y = C_1 + C_2e^x$

7. $y = C_1e^{3x} + C_2e^{-2x}$ **9.** $y = C_1e^{x/2} + C_2e^{-2x}$

11. $y = C_1e^{-3x} + C_2xe^{-3x}$ **13.** $y = C_1e^{x/4} + C_2xe^{x/4}$

15. $y = C_1 \sin x + C_2 \cos x$ **17.** $y = C_1e^{3x} + C_2e^{-3x}$

19. $y = e^x\left(C_1 \sin\sqrt{3}x + C_2 \cos\sqrt{3}x\right)$

21. $y = C_1e^{(3+\sqrt{5})x/2} + C_2e^{(3-\sqrt{5})x/2}$

23. $y = e^{2x/3}\left(C_1 \sin \dfrac{\sqrt{7}x}{3} + C_2 \cos \dfrac{\sqrt{7}x}{3}\right)$

25. $y = C_1e^x + C_2e^{-x} + C_3 \sin x + C_4 \cos x$

27. $y = C_1e^x + C_2e^{2x} + C_3e^{3x}$

29. $y = C_1e^x + e^x(C_2 \sin 2x + C_3 \cos 2x)$

31. (a) $y = 2 \cos 10x$ (b) $y = \frac{1}{5} \sin 10x$
(c) $y = -\cos 10x + \frac{3}{10} \sin 10x$

33. $y = \frac{1}{11}(e^{6x} + 10e^{-5x})$

35. $y = \dfrac{1}{2} \sin 4x$

37. y'' and y' are not equal for $x < 0$. $y'' > 0$ for all x, but $y' < 0$ for $x < 0$.

39. $y = \frac{1}{2} \cos 4\sqrt{3}t$

41. $y = \dfrac{2}{3} \cos 4\sqrt{3}t - \dfrac{\sqrt{3}}{24} \sin 4\sqrt{3}t$

43. $y = \dfrac{e^{-t/16}}{2}\left(\cos \dfrac{\sqrt{12{,}287}\,t}{16} + \dfrac{\sqrt{12{,}287}}{12{,}287} \sin \dfrac{\sqrt{12{,}287}\,t}{16}\right)$

45. b **46.** d **47.** c **48.** a **49.** Proof

51. False: the general solution is $y = C_1e^{3x} + C_2xe^{3x}$.

52. True **53.** True

54. False: the solution $y = x^2e^x$ requires that $m = 1$ is a triple root of the characteristic equation. Because the characteristic equation is quadratic, $m = 1$ can be at most a double root.

55. Proof **57.** Proof

59. (a) Proof (b) $y = \dfrac{C_1}{x^3} + \dfrac{C_2}{x^2}$

Section 15.4 *(page 1123)*

1. Proof **3.** Proof **5.** $y = C_1e^x + C_2e^{2x} + x + \frac{3}{2}$

7. $y = \cos x + 6 \sin x + x^3 - 6x$

9. $y = C_1 + C_2e^{-2x} + \frac{2}{3}e^x$

11. $y = (C_1 + C_2x)e^{5x} + \frac{3}{8}e^x + \frac{1}{5}$

13. $y = -1 + 2e^{-x} - \cos x - \sin x$

15. $y = \left(C_1 - \dfrac{x}{6}\right)\cos 3x + C_2 \sin 3x$

17. $y = C_1e^x + C_2xe^x + \left(C_3 + \dfrac{2x}{9}\right)e^{-2x}$

19. $y = \left(\dfrac{4}{9} - \dfrac{1}{2}x^2\right)e^{4x} - \dfrac{1}{9}(1 + 3x)e^x$

21. (a) $y_p'' = 0$ and $3y_p = 12$ (b) $y_p = 2$ (c) $y_p = 4$

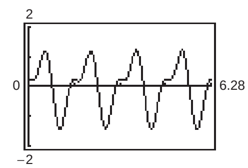
23. $y = (C_1 + \ln|\cos x|)\cos x + (C_2 + x)\sin x$

25. $y = \left(C_1 - \dfrac{x}{2}\right)\cos 2x + \left(C_2 + \dfrac{1}{4}\ln|\sin 2x|\right)\sin 2x$

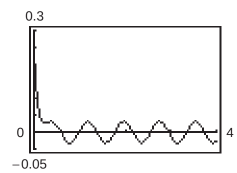
27. $y = (C_1 + C_2x)e^x + \dfrac{x^2e^x}{4}(\ln x^2 - 3)$

29. $q = \frac{3}{25}(e^{-5t} + 5te^{-5t} - \cos 5t)$

31. $y = \frac{1}{4} \cos 8t - \frac{1}{2} \sin 8t + \sin 4t$



33. $y = \left(\frac{9}{32} - \frac{3}{4}t\right)e^{-8t} - \frac{1}{32} \cos 8t$



35. $y = \dfrac{\sqrt{5}}{4} \sin\left(8t - \arctan \dfrac{1}{2}\right)$ **37.** Proof
$= \dfrac{\sqrt{5}}{4} \sin(8t - 0.4636)$

39. $y = C_1x + C_2x \ln x + \frac{2}{3}x(\ln x)^3$

Section 15.5 *(page 1127)*

1. Proof **3.** Proof **5.** Proof

7. $y = a_0\displaystyle\sum_{k=0}^{\infty} \frac{(-3)^k}{2^kk!}x^{2k}$

Interval of convergence: $(-\infty, \infty)$

9. $y = a_0 + a_1\displaystyle\sum_{k=0}^{\infty} \frac{x^{2k+1}}{2^k(k!)(2k + 1)}$

Interval of convergence: $(-\infty, \infty)$

11. $y = a_0\left(1 - \dfrac{x^2}{8} + \dfrac{x^4}{128} - \cdots\right) +$
$a_1\left(x - \dfrac{x^3}{24} + \dfrac{7x^5}{1920} - \cdots\right)$

13. Taylor's Theorem: $y = 2 + \frac{2x}{1!} - \frac{2x^2}{2!} - \frac{10x^3}{3!} + \frac{2x^4}{4!} + \ldots$

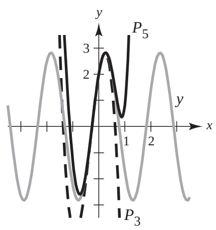
$y\left(\frac{1}{2}\right) \approx 2.547$

Euler's Method: $y\left(\frac{1}{2}\right) \approx 2.672$

15. (a) $y = 2(\cos 3x + \sin 3x)$

(b) $y = 2\left[\sum_{n=0}^{\infty} \frac{(-1)^n(3x)^{2n}}{(2n)!} + \sum_{n=0}^{\infty} \frac{(-1)^n(3x)^{2n+1}}{(2n+1)!}\right]$

(c)



17. $y = 1 - \frac{3x}{1!} + \frac{2x^3}{3!} - \frac{12x^4}{4!} + \frac{16x^6}{6!} - \frac{120x^7}{7!} + \cdot \cdot \cdot$

$y\left(\frac{1}{4}\right) \approx 0.253$

19. Proof **21.** Proof

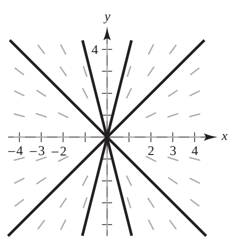
23. $y = a_0 + a_1 x + \frac{a_0}{6}x^3 + \frac{a_1}{12}x^4 + \frac{a_0}{180}x^6 + \frac{a_1}{504}x^7$

Review Exercises for Chapter 15 *(page 1128)*

1. Type: partial
Order: 2

3. Type: ordinary
Order: 2

5. (a)



(b) $y = Cx$

7. b **8.** d **9.** a **10.** c

11. $y = x \ln x^2 + 2x^{3/2} + Cx$ **13.** $y = C(1 - x)^2$

15. $y^2 = x^2 \ln x^2 + Cx^2$

17. $5x^2 + 8xy + 2x + \frac{5}{2}y^2 + 2y = C$

19. $xy - 2xy^3 + x^2 = C$

21. $y = e^x(1 + \tan x) + C \sec x$

23. $x^2 - 2xy - 10x - 3y^2 + 4y = C$ **25.** $y^2 = 2Cx + C^2$

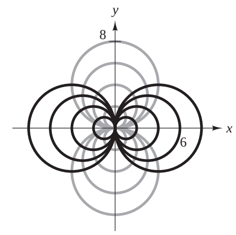
27. $y^2 = x^2 - x + \frac{3}{2} + Ce^{-2x}$ **29.** $\frac{y - x}{1 + xy} = C$

31. $y = \frac{bx^4}{4 - a} + Cx^a$ **33.** $y = 5e^{2x} - e^x$

35. $y = x \ln|x| - 2 + 12x$ **37.** $\ln|1 + y| = (\ln 3)e^{-x}$

39. $y = \frac{3(1 + e^{2x^3+1})}{1 - e^{2x^3+1}}$

41. Family of circles: $x^2 + (y - K)^2 = K^2$



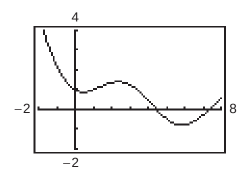
43. (a) $\frac{ds}{dh} = \frac{k}{h}$

$s = k \ln h + C$

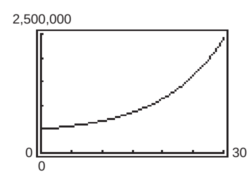
(b) $s = 25 - \frac{13 \ln(h/2)}{\ln 5}$, $2 \le h \le 15$

45. $N = \frac{500}{1 + 4e^{-0.2452t}}$

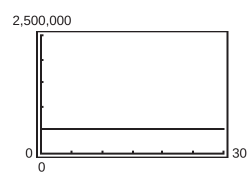
47. $y = \frac{2}{5}(\sin x - 2\cos x) + \frac{9}{5}e^{-x/2}$



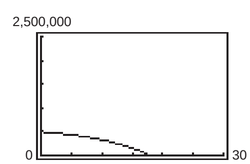
49. (a) Balance increases.



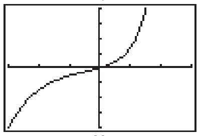
(b) Balance remains $500,000.



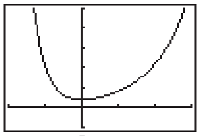
(c) Balance is depleted in 17.9 years.



51. $y = e^{2x} - e^{-x}$



53. $y = \frac{3}{2}e^{x} + \frac{1}{2}e^{-3x}$



55. $y = C_1 \sin x + C_2 \cos x - 5x + x^3$

57. $y = (C_1 + x)\sin x + C_2 \cos x$

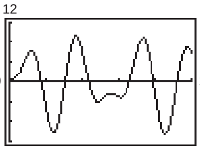
59. $y = \left(C_1 + C_2 x + \frac{1}{3}x^3\right)e^{x}$

61. $y = \frac{11}{5}(2e^{3x} + 3e^{-2x}) - 9$

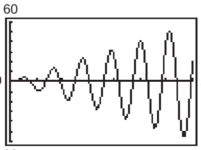
63. $y = \frac{17}{3}\cos 2x - 3\sin 2x + \frac{1}{3}\cos x$

65. $y = \frac{1}{2}\cos\left(2\sqrt{6}\,t\right)$

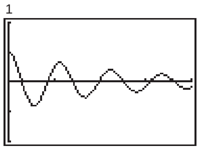
67. (a) (i) $y = \frac{1}{2}\cos 2t + \frac{12\pi}{\pi^2 - 4}\sin 2t + \frac{24}{4 - \pi^2}\sin \pi t$



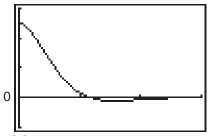
(ii) $y = \frac{1}{2}\left[\left(1 - 6\sqrt{2}t\right)\cos\left(2\sqrt{2}t\right) + 3\sin\left(2\sqrt{2}t\right)\right]$



(iii) $y = \frac{e^{-t/5}}{398}\left[199\cos\frac{\sqrt{199}t}{5} + \sqrt{199}\sin\frac{\sqrt{199}t}{5}\right]$



(iv) $y = \frac{1}{2}e^{-2t}(\cos 2t + \sin 2t)$



(b) The object would come to rest more quickly. It might not oscillate at all, as in part (iv).

(c) The object would oscillate more rapidly.

(d) Part (ii). The amplitude becomes increasingly large.

69. $y = a_0 \sum_{n=0}^{\infty} \frac{x^n}{4^n}$