

Chapter 15

Interacting Populations

Species of animals are introduced or reintroduced into specific habitats for a variety of reasons, both deliberate and accidental. Sometimes such an introduction results in an environmental tragedy in which the alien species threatens or even displaces one or more native species in the region.

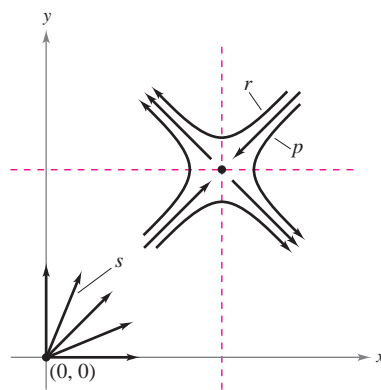
In the 1920s, mathematicians Lotka and Volterra independently developed mathematical models that can represent many of the different ways that two species interact with each other. The common ways in which two species interact are (1) as predator and prey, (2) as complimentary species, and (3) as competing species. For instance, consider a large pond that contains a trout population and a bass population. Although these fishes do eat each others' young, they are usually classified as competing populations because they compete for the same food supply. Let x represent the number of bass, let y represent the number of trout, and let t represent the time (in months). Then, the rates of change of each population can be represented by the following system of differential equations. (In this system, a , b , m , and n are positive constants.)

$$\frac{dx}{dt} = ax - bxy \quad \text{and} \quad \frac{dy}{dt} = my - nxy$$

These equations have many possible solutions that can be obtained by solving the following differential equation.

$$\frac{dx}{dy} = \frac{dx/dt}{dy/dt} = \frac{ax - bxy}{my - nxy}$$

The particular solutions depend on the initial values of x and y and on the values of a , b , m , and n . Graphs of several solutions are shown below. The arrowheads on the graphs denote the directions the populations take over time.



QUESTIONS

1. How would you describe a point of equilibrium for the two populations? Knowing that an equilibrium point must be located where

$$\frac{dx}{dt} = \frac{dy}{dt} = 0$$

find the coordinates of each equilibrium point shown above.

2. For the curves labeled p , r , and s on the graph, describe what is happening to x and y as t increases. Explain this in terms of bass and trout.
3. From which starting points do the bass “win”? From which do the trout “win”?
4. How might you expect the equations and graphs to change if you were studying two species that had a predator/prey relationship rather than a competing relationship?
5. Solve the differential equation $dx/dy = (ax - bxy)/(my - nxy)$ using a technique you learned in Chapter 5. Then assign different values of a , b , m , and n and graph the resulting solutions. Do you obtain graphs similar to those shown above?

The concepts presented here will be explored further in this chapter. For an extension of this application, see the Lab 20 in the lab series that accompanies this text at college.hmco.com.