

**Objective: Approximate the area of a plane region.**

Use upper and lower sums to approximate the area of the region defined by the function  $y = \frac{10}{x^2 + 1}$  on the interval  $[0,2]$ . Use four subintervals with equal width. (Hint: Draw a picture first.)

**ANSWER:**

There are four subintervals ( $n = 4$ ) so the width of each subinterval is equal to  $\frac{1}{2}$ .  $\left(\Delta x = \frac{1}{2}\right)$

$$y = \frac{10}{x^2 + 1}, \Delta x = \frac{1}{2}, n = 4$$

$$S(n) = S(4) = \frac{1}{2} \left[ \frac{10}{1} + \frac{10}{\left(\frac{1}{2}\right)^2 + 1} + \frac{10}{1^2 + 1} + \frac{10}{\left(\frac{3}{2}\right)^2 + 1} \right] \approx 13.0385$$

$$S(n) = S(4) = \frac{1}{2} \left[ \frac{10}{\left(\frac{1}{2}\right)^2 + 1} + \frac{10}{1^2 + 1} + \frac{10}{\left(\frac{3}{2}\right)^2 + 1} + \frac{10}{2^2 + 1} \right] \approx 9.0385$$

$$9.0385 < \text{Area of Region} < 13.0385$$