

Objective: Simplify the derivative of a function using algebra.

Find the derivative of the function $f(x) = \frac{x}{\sqrt{x^4 + 4}}$.

ANSWER:

$$\begin{aligned} f(x) &= \frac{x}{\sqrt{x^4 + 4}} = \frac{x}{(x^4 + 4)^{\frac{1}{2}}} \\ f'(x) &= \frac{(x^4 + 4)^{\frac{1}{2}}(1) - \left[\frac{1}{2}x(x^4 + 4)^{-\frac{1}{2}}(4x^3) \right]}{\left((x^4 + 4)^{\frac{1}{2}} \right)^2} \\ &= \frac{(x^4 + 4)^{\frac{1}{2}} - \left[2x^4(x^4 + 4)^{-\frac{1}{2}} \right]}{x^4 + 4} \\ &= \frac{(x^4 + 4)^{\frac{1}{2}} - 2x^4(x^4 + 4)^{-\frac{1}{2}}}{x^4 + 4} \\ &= \frac{(x^4 + 4)^{\frac{1}{2}} \left[(x^4 + 4) - 2x^4 \right]}{x^4 + 4} \\ &= (x^4 + 4)^{-\frac{3}{2}}(x^4 + 4 - 2x^4) \\ &= \frac{x^4 + 4 - 2x^4}{(x^4 + 4)^{\frac{3}{2}}} \\ &= \frac{4 - x^4}{(x^4 + 4)^{\frac{3}{2}}} \end{aligned}$$