Objective: **Use the limit definition to find the derivative of a function.**

Find the derivative of the function $f(x) = -3x^2 + 1$.

ANSWER:

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-3(x+\Delta x)^2 + 1 - (-3x^2 + 1)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-3(x+\Delta x)(x+\Delta x) + 1 + 3x^2 - 1}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-3(x^2 + 2x\Delta x + (\Delta x)^2) + 3x^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-3x^2 - 6x\Delta x - 3(\Delta x)^2 + 3x^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-6x\Delta x - 3(\Delta x)^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-3\Delta x(2x + 1\Delta x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} -3(2x + 1\Delta x)$$

$$= \lim_{\Delta x \to 0} -6x - 3\Delta x$$

$$= \lim_{\Delta x \to 0} -6x - 3(0)$$

$$= \lim_{\Delta x \to 0} -6x = -6x$$