

EXPLORATION 9.7 **Lattices and Geometric Structure**

Crystals have fascinated humans for many thousands of years. Throughout the ages, and even today, many people have attributed certain powers to crystals. A question that puzzled people for thousands of years was, “Why do only a few kinds of rocks have these orderly shapes?” For example, the basalt rocks shown in the photo have formed in a hexagonal pattern. The origin of the word *crystal* gives a clue to early hypotheses—the word comes from the Greek word for ice. By the seventeenth century, even before modern atomic theory, scientists were hypothesizing that the shapes of crystals were due to an orderly, patterned internal structure. Some suggested that crystals are made of stacks of tiny spheres or bricks that represented the basic particles of the structure, whatever those might be.⁴ Sphere packings and bricks are still important models for crystal structure today.

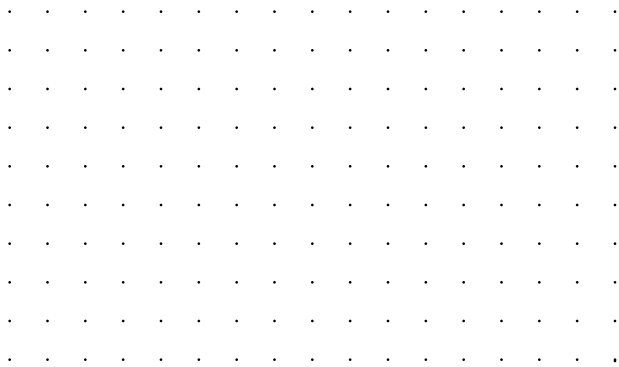
This notion of an orderly array connects to the mathematical concept of lattices. A one-dimensional lattice is essentially a set of dots that are evenly spaced along a line. One-dimensional lattices are not terribly interesting, but two-dimensional lattices both are interesting mathematically and have applications beyond mathematics. A two-dimensional lattice may be thought of as a repetition of row after row of one-dimensional lattices. The dots on each row can be directly under each other, or the rows can be shifted so that the dots in every other row have been moved (translated) horizontally. This creates many possibilities.

This exploration comes from a question that relates to ideas in physics. Imagine that each dot occupies all the space (on the paper) that is closer to it than to any other dot. Imagine drawing a fence to denote the space that is closer to a given

dot than to any other dot; that is, the fence would be the set of points in “no man’s land.” Each one of these regions bordered by a fence is called a Dirichlet domain.

PART 1

1. *Square lattice.*
 - a. Determine the Dirichlet domains for the lattice on page 243.

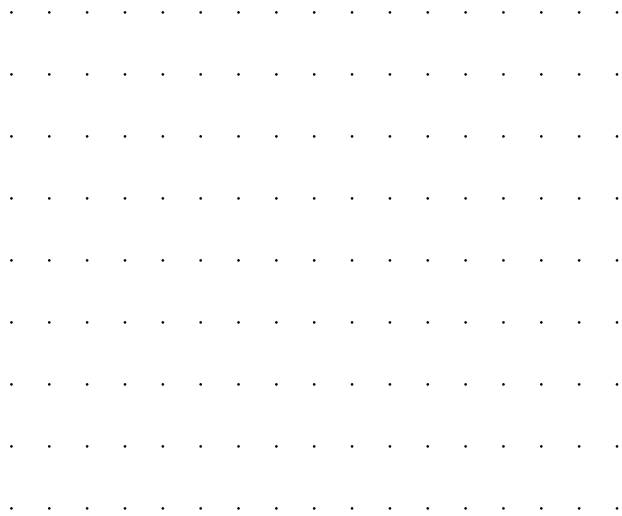


- b. Describe how you solved the problem.
 - c. Predict the shape of the fence for the lattice shown in Step 2—in that case, the dots are still below each other, but the vertical distance between two adjacent dots is greater than the horizontal distance between two adjacent dots. Explain the reasoning behind your prediction.

⁴Marjorie Senechal, *On the Shoulders of Giants*, p. 155.

2. *Rectangular lattice.*

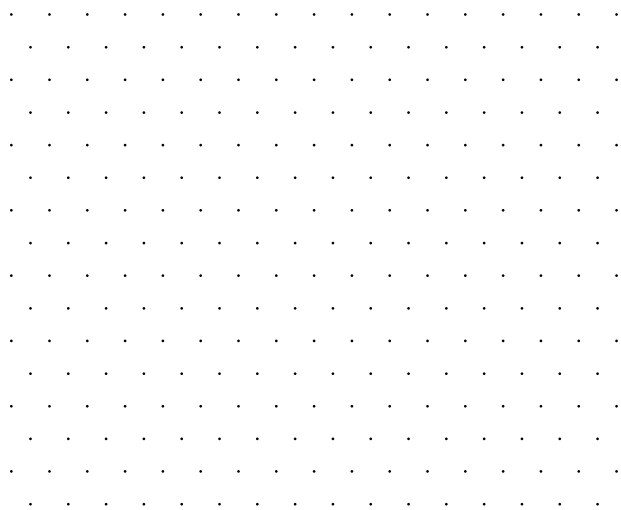
a. Determine the Dirichlet domains for the lattice below.



b. Describe how you solved the problem.

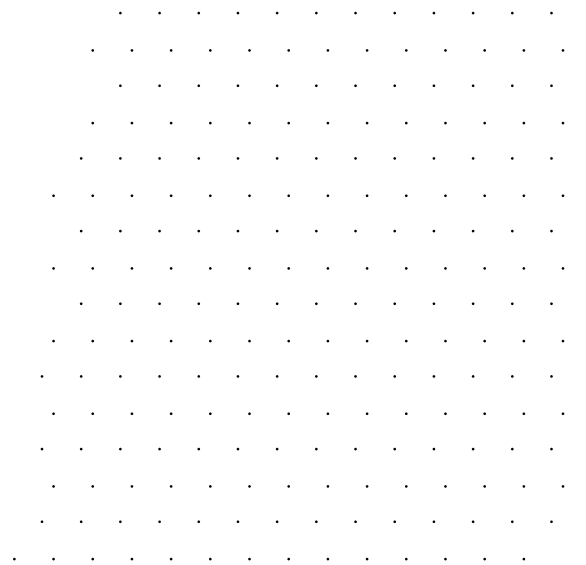
c. Predict the shape of the fence for the lattice shown in Step 3—in that case, every other row of dots has been translated horizontally. Explain the reasoning behind your prediction.

3. *Staggered lattice.* Determine the Dirichlet domains for the lattice below. Describe how you solved the problem.



PART 2

1. What other possible lattices might produce different kinds of regions? Write down your thoughts, and then discuss your ideas with your partner(s) and determine how you could create those lattices and find new Dirichlet domains.
2. Describe your findings.
3. *Another staggered lattice.* Determine the Dirichlet domains for the lattice on page 245. Describe how you solved the problem.



Looking Back on Exploration 9.7

1. What generalizations can you make about Dirichlet domains?
2. Will all Dirichlet domains have symmetry? If so, will they have the same kind of symmetry? If not, why not?