EXPLORATION 8.9 **Exploring Line Segments in Triangles**

In high school geometry, as you may recall, you spent more time studying triangles than any other figure, and theorems developed with triangles — about congruence, similarity, the sum of interior angles, and other topics — were extended to other polygons. Even though it is the simplest polygon (simply take three noncollinear points and connect them in turn), it is the most studied. The triangle is a powerful shape, a central figure in quilting and many other forms of art and construction. It is at the heart of the geodesic dome, which Buckminster Fuller invented. Fuller once remarked: "A triangle is the most economical and natural path a structure can take. Triangle is structure and structure is triangle. Period. It's what the whole Universe is based on."[2] Fuller sounds very much like a Pythagorean!

The following explorations are inspired by "what-if" games (a favorite game of young children, poets, scientists, inventors, and other curious people). They are designed to invite different kinds of ways in which people come to understand
ideas — deductively, inductively, and intuitively.

PART 1: The three medians of a triangle

1. What would happen if we drew the three *medians* of a triangle? How might you design your exploration of this question? Write down your thoughts about what would happen and how you might explore this question.

2. Compare notes with your partner(s).

3. Design a plan and then describe your plan so that your instructor can see your design and your work on the left side and your reflections and analyses on the right side.

4. Summarize your findings as conjectures.

5. Explore one or more of these questions. In each case, first state your initial response, then design your experiment, do the work, and report your findings.

- **a.** The point at which all three medians intersect is called the *centroid* of the triangle. The centroid divides each of the medians into two shorter line segments. What relationships can you find among the lengths of these line segments?
- **b.** Can other polygons have medians, or only triangles?

PART 2: The three angle bisectors of a triangle

1. Draw the three angle bisectors of a number of triangles. As before note your reflections and analyses on the right side.

2. Summarize your findings as conjectures.

3. Explore one or more of these questions. In each case, first state your initial response, then design your experiment, do the work, and report your findings.
   a. The point at which all three angle bisectors intersect is called the *incenter* of the triangle. If we find the centroid and incenter of a triangle, will they be the same point in some triangles? in all triangles? never?

   b. Under what circumstances will the angle bisector also be a median?

   c. Can we generalize our findings about incenters to other polygons?

PART 3: The three perpendicular bisectors of a triangle

1. Draw the three perpendicular bisectors of a number of triangles. As before, note your reflections and analyses on the right side.

2. Summarize your findings as conjectures.

3. Explore one or more of these questions. In each case, first state your initial response, then design your experiment, do the work, and report your findings.
   a. The point at which all three perpendicular bisectors intersect is called the *circumcenter* of the triangle. If we find the centroid, incenter, and circumcenter of a scalene triangle, they will be different points; are they collinear or noncollinear?

   b. Under what circumstances will the perpendicular bisector also be a median?

   c. Can we generalize our findings about circumcenters to other polygons?

PART 4: A question of utility

Each of these three points — centroid, incenter, and circumcenter — has a useful role. One of the points (incenter) is the center of the circle that can be inscribed in the triangle (see the figure on the left below). Another (circumcenter) is the center of the circle that can be circumscribed about the triangle (see the figure on the right below). Yet another (centroid) is the center of gravity of the triangle. That is, if you made a triangle from heavy material (posterboard, wood, or metal), the triangle would balance at this point (see the figure in the middle below). Which will be which? Why?

[1] This exploration is adapted from a talk by Glenda Lappan entitled "The Connected Mathematics Project: Reaching All Kids Through Interesting Problems and New Instructional Strategies," given at the New Hampshire ATMNE meeting in Plymouth, New Hampshire, on April 1, 1994.
[2] Cited in Julian Weissglass, *Exploring Elementary Mathematics* (Dubuque, Iowa: Kendall Hunt Publishing, 1990), p. 161.