

EXPLORATION 5.15 **Patterns in Repeating Decimals**

When we convert some fractions to decimals, we find that some “terminate” and some “repeat.” For example, convert $1/11$ and $1/16$ to decimals with your calculator.

$$1/16 = 0.0625 \text{ but } 1/11 = 0.09090909090909 \dots$$

Rather than writing the “dot dot dot,” we place a bar over the portion that repeats, and we represent $1/11$ as $0.\overline{09}$.

How would you represent $2/3$ and $7/15$?

Part 1: Predicting the next number in a sequence

1. We used a calculator above to determine the value of $1/11$.
 - a. Predict the value of $2/11$ and explain your reasoning.

Now use the calculator to determine the value of $2/11$.

If your prediction was correct, great. If not, look back at your reasoning and see if you can modify it (like finding a bug in a computer program). Describe your insights.

- b. Predict the value of $3/11$ and explain your reasoning. Then determine the value of $3/11$. If your prediction was correct, great. If not, analyze your thinking.
 - c. Describe the pattern you see in words. If you do not see a pattern, determine the value of $4/11$, $5/11$, and so on until you do see a pattern.
 - d. Predict the value of $9/11$ and explain your reasoning. Then determine the value of $9/11$. If your prediction was correct, great. If not, analyze your thinking.
 - e. Predict the value of $10/11$ and explain your reasoning. Then determine the value of $10/11$. If your prediction was correct, great. If not, analyze your thinking.
2. Use a calculator to determine the value of $1/15$.
 - a. Predict the value of $2/15$ and explain your reasoning. Then determine the value of $2/15$. If your prediction was correct, great. If not, analyze your thinking.
 - b. Predict the value of $3/15$ and explain your reasoning. Then determine the value of $3/15$. If your prediction was correct, great. If not, analyze your thinking.

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- c. Continue this process until you see a pattern. Then describe the pattern you see in words.
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5. Now we will look at the set of fractions whose denominator is 101.
 - a. Based on your work, can you predict the value of $1/101$? If so, explain the reasoning behind your hypothesis. Then determine the value of $1/101$ using pencil and paper.
 - b. Predict the value of $2/101$ and explain your reasoning. Then determine the value of $2/101$. If your prediction was correct, great. If not, analyze your thinking.
 - c. Continue this process until you see a pattern. Then describe the pattern you see in words.

Part 2: When will a decimal terminate and when will it repeat?

Now that you have some experience with converting fractions to decimals, we will investigate a question that fascinated mathematicians several hundred years ago: Can we predict whether the decimal equivalent of a fraction will terminate or repeat?

1. Write down your present thoughts as hypotheses. For example, if the denominator is an even number, its decimal equivalent will terminate, and so on.
2.
 - a. With your partners, determine a plan to test your hypotheses.
 - b. Meet with another group and share your ideas.
 - c. Describe your plan.
3. Collect your data. That is, select a fraction and predict whether it will terminate or repeat. Then do the computation. At the right, note any insights from your work. Your sheet should look something like this:

Number	Prediction	Analysis/insight
$5/13$	Repeat because odd number	It repeats.

4. Write down your current hypotheses, and justify them.