EXPLORATION 5.12 **Understanding and Creating Division Algorithms**[11]

Part 1: Invert and multiply

Most of you were taught the famous invert and multiply algorithm for dividing a fraction by a fraction. The purpose of this exploration is for you to understand why it works.

Directions

In each of the problems below, you need to:

a. Represent the story with a diagram.

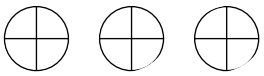
b. Explain how you can determine the quotient from the diagram. (*Note:* This requires explaining *why*, not just *what*.)

c. Write a number sentence involving division and connect each of its numbers to the problem.

d. Write a number sentence involving multiplication and connect each of its numbers to the problem.

Example

Jerome had 3 pounds of candy and decided to make 1/4-pound bags to give away. How many bags could he make with this candy?

a. 

b. Each circle represents 1 pound of candy. I divided each pound into 4 regions (to make 1/4-pound bags). I solve the problem by counting the number of 1/4ths in the 3 pounds.

c. Number sentence with division: $3 \div 1/4$
The 3 represents the whole: 3 pounds.
The 1/4 represents the size of the part I am repeatedly taking away.

d. Number sentence with multiplication: $3 \cdot 4$
The 4 represents how many 1/4-pound bags per pound.
The 3 times 4 represents 4 bags per pound times 3 pounds.

Problems

1. A microscopic organism undertakes a journey of 2 inches. Each day the organism travels 1/5 inch. How many days will it take to finish the journey?

2. Pete's Painting Partnership has contracted to paint 4 houses. Each day the team can paint 2/3 of a house. How long will it take to finish all 4 houses?

3. Eno has 4 ounces of medicine. Each dose is 1 1/3 ounces. How many doses are there in the bottle?

Looking Back

How would you justify the invert and multiply algorithm now?

Part 2: Brahmagupta's algorithm

The famous Indian mathematician Brahmagupta described an algorithm for dividing fractions that is actually easier to understand, in terms of *why* it works, than the traditional invert and multiply algorithm. The purpose of this exploration is to help you to understand this algorithm and why it works.

For each of the problems below, first represent the problem with a diagram and then determine the answer from the diagram.

1. Josie's Jammers have adopted a 4-mile stretch of highway to keep clean. Each afternoon they pick up trash. If they can clean 2/3 of a mile per day, how many days will it take to clean the whole section?

2. Chien has 2/3 of a gallon of gasoline, and each time he mows the lawn, he uses 1/6 of a gallon. How many times can he mow the lawn before buying more gasoline?

3. Jack has $7\frac{1}{2}$ pounds of dog food. Each day his dog eats $1\frac{1}{4}$ pounds. How many days' worth of dog food does he have?

4. Rita has $3\frac{3}{4}$ ounces of perfume and wants to sell the perfume in 3/8-ounce bottles. How many bottles of perfume can she make?

Looking Back

Look at your solutions to each of the problems above.

1. What do they all have in common?

2. Looking at the commonalities, can you describe a rule that would allow you to solve any division problem? That is, what did you do to your whole so that you could repeatedly subtract the part? A hint is given at the end of Exploration 5.13.

[11] This exploration has benefited from the influence of Ellen Davidson and Jim Hammesman at Education Development Center.