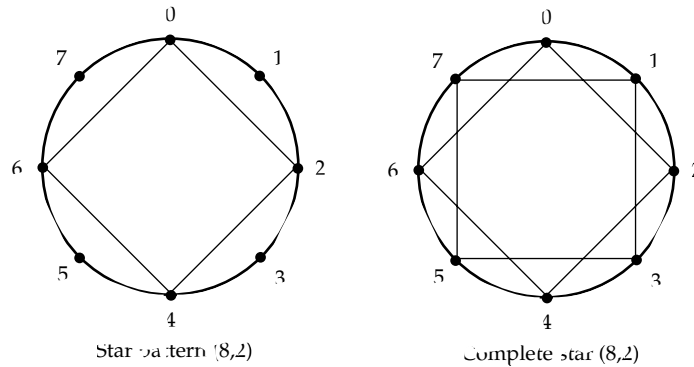


EXPLORATION 4.5 Paths and Orbits in Star Patterns

We have found that when we make star patterns, sometimes we will connect all the points on the circle without lifting our pen or pencil and sometimes we will not. From one perspective, if not all the points have been used, the star is incomplete. For example, the figure on the left below shows star $(8, 2)$, which is not complete. However, if we move over one point (to point 1) and connect every other dot, we now have a “complete” star, as shown on the right.

**Paths**

To use language that other mathematicians have used, we can say that to complete star $(8, 2)$, we have to make two **paths**. A path can be defined as the figure made by starting at one point and continuing until you reach that point again. Thus, the complete star $(8, 2)$ consists of 2 paths.

1. Can you predict the number of paths it will take to make a complete star? That is, how many paths does star (x, n) have?

Suggestion: Make some stars with different x 's and different n 's. Note the number of paths it takes to make a complete star. Do this until you feel that you can predict the number of paths in the complete star knowing only x and n . Note that there are *universal hypotheses*, those that will work for all x 's and n 's, and there are *specific hypotheses* (an example of a specific hypothesis is, “All stars of the form $(7, n)$ will have exactly 1 path”).

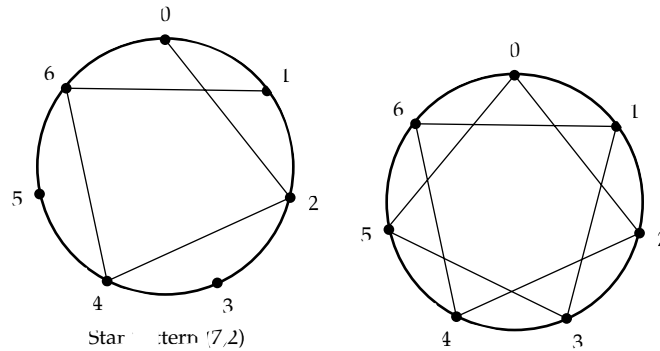
2. If a star has more than one path, must each path in that star be the same length? What do you think?
3. If a star has more than one path, must each path in that star contain the same number of points? What do you think? Is this question essentially the same as the one asked in Step 2? Why or why not?

Orbits

We can also speak of the number of *orbits* in a star pattern.

4. Before we begin, what do you think an orbit will mean? For example, look back to your previous work. How many orbits are there in star $(8, 2)$? in star $(7, 2)$? Do this before reading on.

One simple definition of **orbit** is a trip around the circle in which you either “pass” your starting point or land “at” your starting point. Thus star $(8, 2)$ has one orbit because when you end up at the starting point, you have gone around the circle once. However, star $(7, 2)$ has 2 orbits because you must go around the circle 2 times before you arrive at the original starting point. The figures below demonstrate this fact. If we designate 0 as the starting point, our path goes from 0 to 2 to 4 to 6 to 1. At this point, we have gone around the circle once (actually slightly more than once). If we continue, we go from 1 to 3 to 5 to 0. We now have gone around the circle twice and have arrived at our starting point. Therefore, star $(7, 2)$ has 2 orbits.



5. Can you predict the number of orbits in a star pattern? That is, how many orbits will star (x, n) have?

Suggestion: Make a number of star patterns with different x 's and n 's. Note the number of orbits in each star. Do this until you feel you can predict the number of orbits in any star knowing only x and n .