APPENDIX B

LOGO

Your mathematics education differs in many ways from that of your grand-parents, and I firmly believe that the mathematics education of children 20 years from now will differ even more. One of the reasons for this profound difference will be technology. The potential for using calculators and computers to help people learn mathematics is extraordinary.

Computers are almost everywhere today, but this was not always so. Although I am not so old (I started teaching in 1973), my first substantive interaction with computers was in 1984, when I bought one for graduate school. I was terrified! But now I don't know how I would teach without them. However, it's a different story with my children. When they were still preschoolers, I bought them some software and was amazed at how quickly they caught on. I was even more amazed when I came into my home office one day and found 4-year-old Emily playing on my computer by herself. She had figured out how to turn it on, get into her program, quit her program, and play with my programs! Many of your students will be as comfortable with computers as my children are, and most children will quickly learn to use computers as long as they sense your enthusiasm and confidence.

It has been over 20 years since computers first started finding their way into schools. While they have had many positive effects, there have been as many misuses and abuses of computers as there have been positive uses. Toward this end, NCTM issued a position statement on the use of computers in the learning and teaching of mathematics. You will examine the whole statement in your Methods course later. At this point, we will focus on one part that is relevant to this course:

> Teachers should plan for students' use of technology in both learning and doing mathematics. A development of ideas is to be made with the transition from concrete experiences to abstract mathematical ideas,

focusing on the exploration and discovery of new mathematical concepts, and problem solving processes.[1]

LOGO

The computer language most often used in elementary schools and the one I find the most exciting is called LOGO. It was developed in the 1960s by a team of researchers at MIT under the direction of Seymour Papert. Papert was strongly influenced by Jean Piaget, a man about whom you will probably read more in your education courses than about any other theorist. One of Piaget's fascinations was how children are able "to build complex knowledge out of simple components."[2] Papert wanted to create a computer language that would reflect Piaget's views of how children learn.

Work on LOGO began with a mechanical device, called a turtle, that children could move around by issuing commands through a control box. The turtle had a pen under its center so that children could literally see the results of their commands. Ultimately, these ideas moved from a mechanical turtle to a simulated turtle that appears on a computer screen. There are many resources that give both the history of LOGO and how to use it, and the interested reader is encouraged to consult them. For now, let's get started.

Getting Started

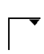
If you have never worked with LOGO before, open the program and type the following commands:[3]

```
FD 50 RT 90 FD 50 RT 90
```

You can press RETURN or ENTER after each command, or you can type all four commands and then press RETURN.

Let us decode these commands. FD 50 means "go forward 50 units." Different LOGO programs have different units, so FD 50 will not necessarily make congruent line segments with different versions. On the other hand, RT 90 will have the same effect regardless of the LOGO program, because it means "turn right 90 degrees," and all LOGO programs accept 360 degrees as one complete revolution.

You should see this on the screen, with the turtle at the top and pointing down.



On your own, enter the commands you think will complete this figure to make a square. You may want to press HT at the end so that the computer will hide the turtle.

If you added the following commands, you will see a square:

```
FD 50 RT 90 FD 50 RT 90
```

Table B.1 gives the basic commands that we will use in this course, although your instructor may add more.[4]

[1] From the NCTM Position Statement "The Use of Technology in the Learning and Teaching of Mathematics," in the *1995–96 Handbook: NCTM Goals, Leaders, and Positions* (Reston, Va.: NCTM, 1995), p. 24.

[2] Earl Babbie, *Apple Logo for Teachers* (Belmont, Calif.: Wadsworth, 1984), p. 4.

[3] You must have a space between these commands. For example, you will receive an error message if you type FD50. Sometimes a space is essential, sometimes it is optional, and sometimes no space is essential.

[4] Some LOGO versions will have slightly different commands.

TABLE B.1

Function	Command	Abbreviation
Move the turtle forward	FORWARD	FD
Move the turtle backward	BACK	BK
Turn the turtle to its right	RIGHT	RT
Turn the turtle to its left	LEFT	LT
Lift the pen so that the turtle will not leave a trail	PEN UP	PU
Put the pen back down	PEN DOWN	PD
Hide the turtle so that you can see the picture	HIDE TURTLE	HT
Show the turtle so that you can see where it is	SHOW TURTLE	ST
Return the turtle to the center of the screen	HOME	
Clear the screen	CLEAR SCREEN	CS

The REPEAT Command

We have just seen how to make a square by typing in eight separate commands The square is one of the most basic geometric figures, so imagine how many commands it would take to have the turtle make other geometric figures. Fortunately, there is a command called REPEAT that can greatly reduce the number of commands needed. Let us examine how it works.

If we analyze our eight commands to make a square, we can see that we are simply writing “FD 50 RT 90” four times. Enter the following command into your computer now.[5]

```
REPEAT 4 [FD 50 RT 90]
```

In one sense, the REPEAT command is like the distributive property—for example, 4[5 + 3]. This kind of repetition is called iteration, which is one of the big ideas of programming languages.

Defining Procedures

Now make another square, this one 80 units on each side.

As you may have found, you could simply type

```
REPEAT 4[FD 80 RT 90]
```

There are two procedures that can make our work in LOGO even simpler First, any series of operations that we want to use several times can be named using the command “TO.”

Enter the following:

```
TO SQUARE
REPEAT 4 [FD 80 RT 90]
END
```

Second, the “END” tells the computer that the end of this procedure has been reached. You need END when you have a number of procedures in a row Without it, the computer has no way of knowing when you are finished with one procedure. Some current software versions of LOGO will automatically type in “END” when you are finished.

[5]You must leave a space between REPEAT and 4. However, the space between 4 and the open bracket [is optional.

Just as we save computer files, we can save a LOGO program by giving it a name. For example, if you want to make this square tomorrow, you won't have to type REPEAT 4 [FD 80 RT 90]. However, if we want a square of a different size, we need to delete 80 and insert our new length. The next procedure addresses this need.

Defining Procedures with Variables

The following procedure uses the concept of variable and defines a general procedure for making any square.[6] Type the following program into your computer:

```
TO SQUARE :S
REPEAT 4 [FD :S RT 90]
END
```

To make a square with side 95 units in length, we simply enter SQUARE 95. Try it.

At this point, you have enough information to work with all the LOGO material in this appendix. The following Investigation gives you a little more practice, which most people need before launching out on their own.

INVESTIGATION B.1 Making Rectangles

How would you define a LOGO procedure to draw a rectangle whose length is 80 units and width is 40 units? Think and then read on. . . .

DISCUSSION

Using our knowledge of LOGO and geometry, we find that the following commands will make one-half of a rectangle:

```
FD 40 RT 90 FD 80 RT 90
```

Thus, we can make a complete rectangle by typing

```
REPEAT 2 [FD 40 RT 90 FD 80 RT 90]
```

We can define a procedure for making rectangles by using the idea of variables. In contrast to the SQUARE procedure, we need two variables to define a rectangle: L for length and W for width (some people prefer L1 and L2).

```
TO RECTANGLE :L :W
REPEAT 2[FD :L RT 90 FD :W RT 90]
END
```

We can then make the 80-unit by 40-unit rectangle by typing RECTANGLE 80 40.

Some people prefer to define rectangle as RECT because it's a shorter command to type.

Now use this knowledge to make Figure B.1.

[6] In this case, there must be a space between SQUARE, and :, and there must be no space between : and S.

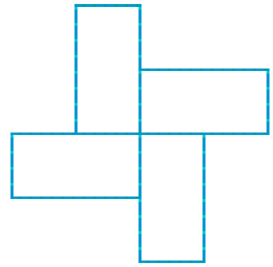


FIGURE **B.1**

This can be made by the following command:

```
REPEAT 4[RECTANGLE 40 80 RT 90]
```

INVESTIGATION **B.2** Making Geometric Figures

Can you make Figure B.2? Think and then read on. . . .

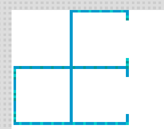


FIGURE **B.2**

DISCUSSION

This figure exemplifies another positive aspect of LOGO (and many other computer languages): It honors the notion that not everyone thinks in the same way. Have you ever had a teacher who showed his or her way to solve a problem and insisted that you do it that way? I had several such teachers, and it almost ruined me!

Below are three different ways to make the figure above. There are many others, too!

```
REPEAT 4 [FD 40 RT 90]
REPEAT 4 [FD 40 LT 90] FD 40
REPEAT 4 [FD 40 RT 90]

RECTANGLE 40 80
LT 90 BK 80
RECTANGLE 40 80

RECTANGLE 80 40
LT 90 BK 40
RECTANGLE 80 40
```

LOGO's Interactive Nature

One of the best aspects of working with LOGO is that students get immediate feedback. If you make a mistake in writing the program, the computer tells you. For example, with LOGO you either get feedback like "I don't know how to

FD50," or the screen shows the figure that is made from your directions. In either case, you can analyze what you did wrong and fix it as opposed to getting a poor grade, telling you that you did it wrong.

In this introduction, it is hoped that not only have you learned about LOGO, but you have also learned a little geometry. In order to solve "good" problems and make figures using LOGO, students must apply their knowledge of geometry and find underlying connections in geometry. If my goals for your work with LOGO are realized, in working with LOGO, you will see the following outcomes in yourself:

- You will be more positively disposed toward mathematics.
- You will learn more about geometry, have a better understanding of certain geometric concepts, and see more connections among concepts.
- Your problem-solving toolbox will be stronger.
- Your logical and analytical thinking abilities will be enhanced.
- Your spatial visualization abilities will be enhanced.

Exploring LOGO

Below are seven Explorations that will enable you to use LOGO to apply your understanding of geometry and to refine your ability to make and test predictions.

EXPLORATION B.1 Making Letters

1. Each of the following sets of commands will create one letter of the alphabet.

a. Predict the letter that will occur, and then type the commands to find out.

```
FD 80 RT 90 FD 40 BK 80
FD 40 BK 80 FD 40 RT 90 FD 40 LT 90 FD 40 BK 80
RT 20 FD 80 LT 40 BK 80 FD 40 LT 70 FD 28
FD 80 LT 20 BK 85 RT 40 FD 85 LT 20 BK 85
```

b. At this point, you may want to define each procedure— that is, TO "LETTER" for each letter.

2. Now use PU, PD, and HOME to spell a word with these four letters on your screen.

The HOME position of the turtle is in the center of your screen. To move the turtle so that you can spell this word, your first command will need to be something like[7]

```
PU LT 90 FD 120 RT 90 PD
```

If we look at the commands in turn, the computer "hears" this:

[7] The actual numbers will vary depending on which version of LOGO you are using.

PU	Lift the pen up.
LT 90	Turn 90 degrees to the left.
FD 120	Move forward 120.
RT 90	Turn 90 degrees to the right (that is, return to the initial heading).
PD	Put the pen down (so that you are ready to work).

3. As you may have realized, not all the letters are the same height, and the procedures used to define the letters did not have them all beginning at the bottom of the letter. Fix the procedures used to define each letter so that all the letters are the same size and all begin at the bottom of the letter. You will be finished when you can get the computer to print your word so that it appears just as it would on a typewriter.

4. Spell another word.

EXPLORATION B.2 **Triangles**

1. Commonalities among all triangle procedures. The following program makes an equilateral triangle:

```
REPEAT 3 [FD 50 RT 120]
```

Most people are at least initially puzzled by the fact that we make an equilateral triangle (whose angles are all 60 degrees) by telling the turtle to move forward and then right 120 degrees three times.

a. Can you explain why this is so? Discuss your ideas in your group and write your first draft of your response.

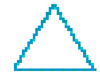
b. After the class discussion, define and execute the following procedures:

```
TO TRI1 FD 50 RT 75 FD 60 RT 146 FD 86 RT 139 FD 40 HT END
TO TRI2 FD 50 RT 100 FD 60 RT 135 FD 70 RT 125 END
TO TRI3 FD 50 RT 90 FD 120 RT 157 FD 130 HT END
```

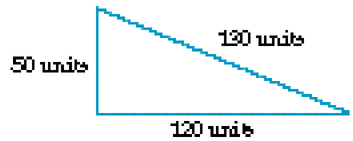
c. Now try to explain the connection between the procedure necessary to make triangles with LOGO and the fact that the sum of the angles of any triangle is 180 degrees.

2. Making triangles that appear in the classic position.

a. Modify the REPEAT 3 [FD 50 RT 120] procedure to produce an equilateral triangle with the base parallel to the bottom of the page.



3. Understanding right triangles. Now let us examine how we can use knowledge of geometry to make right triangles. Make the following triangle on your computer:



EXPLORATION B.3 **Investigating Parallelograms**

PART 1: Properties of parallelograms

This exploration both requires you to apply your understanding of quadrilaterals and will result in a deeper understanding of the properties of quadrilaterals.

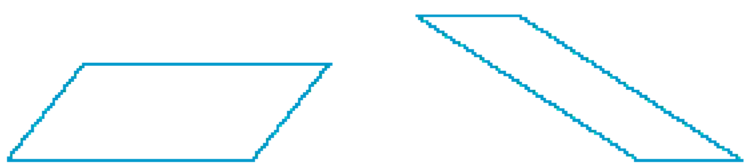
1. Construct several parallelograms with LOGO. Save your procedures— for example, PARA1, PARA2, and so on. Compare your procedures with your partner(s).

2. A parallelogram is defined as a quadrilateral in which both pairs of opposite sides are parallel. What additional properties do you think all parallelograms have, based on your observations on the results of your group's procedures?

3. Write a procedure that will enable you to make any parallelogram. *Note:* First you must think of how many variables you need. For example, a square needed only one variable, whereas a rectangle needed two variables (L for length and W for width).

PART 2: Predicting outcomes

In this part, you will try to make predictions and then generalizations. Given the initial variables, can you predict the shape and orientation of the resultant parallelogram? For example, the two parallelograms below are congruent but in different orientations.



1. Using the following procedure, make a variety of parallelograms.

```
TO PARA :M :N :A
REPEAT 2 [FD :M RT :A FD :N RT 180 - :A]
END
```

a. Sketch the relative shape and orientation of the parallelogram that will appear. That is, predict the shape and orientation.

b. Run the procedure to check your prediction.

c. If your prediction was close, fine. If not, analyze your prediction and what appeared on the screen and describe what you learned so that your next prediction may be more accurate.

2. Make up examples of your own to further explore the relationships between the lengths of the sides and angles and the resulting parallelogam.

3. Describe hypotheses that emerged from your explorations. Justify them.

4. What kind of quadrilaterals can be made with the PARA procedure? Be as specific as possible. For example, when the first two numbers after PARA are equal and the third number is 90, a square will be made.

EXPLORATION B.4 **Regular Polygons**

A regular polygon is defined as a convex polygon in which all sides have the same length and all angles have the same measure. We have learned how to use LOGO to make a regular triangle and a regular quadrilateral, more commonly known as an equilateral triangle and a square.

1. Write a program that can be used to make a regular pentagon and then a regular hexagon. Describe what you learned from this step.

2. Now write a procedure that will make a regular polygon with *n* sides. Explain why your procedure works as if you were talking to someone who missed this class.

EXPLORATION B.5 **Circles**

The following investigation is one of my favorites.

1. Develop a procedure that will make a circle. Summarize your rationale / thinking.

2. Enter the procedure given by your instructor. Why does it work?

EXPLORATION B.6 **Polygons and Stars**

Earlier, you developed a procedure that will create regular polygons. Below is another one.

```
TO POLY :N :S :A
REPEAT :N [FD :S RT :A]
END
```

Now experiment with different values for S, A, and N and see what you can find.

1. Summarize your findings and then compare notes with your partner(s).

2. Write a set of rules that will enable the reader to predict which commands will produce polygons, which will produce stars, and which will produce circles. For those commands that produce stars, write directions for predicting the number of points on the star; for example, POLY 9 60 200 will make a 9-pointed star.

3. In some cases, different commands produce the same star. Explain why.

EXPLORATION B.7 **Polyspirals**

The following famous LOGO program produces polygonal spirals that often look like flowers. For example, try POLYSPI 20 190 HT.

```
TO POLYSPI :L :T
IF :L > 300
 [STOP]
FD :L
RT :T
POLYSPI (:L + 3) :T
END
```

1. Experiment with different values of L and T. Describe your findings. Explain them.

2. Explain what roles the "300" and the "3" play.

EXERCISES

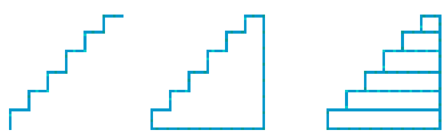
1. Complete this procedure so that it makes a trapezoid:

LT 90 FD 100 RT 43 FD 50

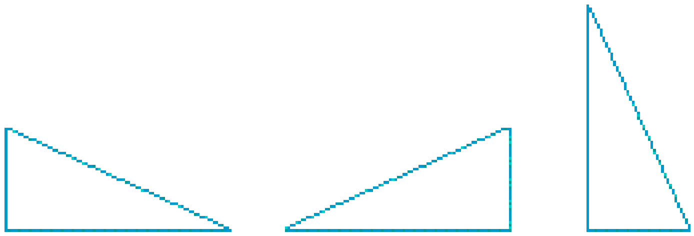
2. Complete this procedure so that it makes a parallelogram:

LT 90 FD 80 RT 45 FD 70

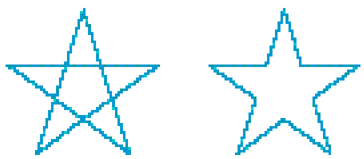
3. Make the following staircases.



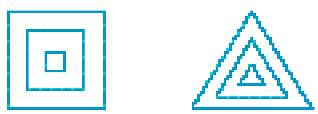
4. Make the following right triangle in different orientations.



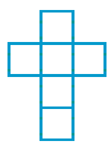
5. Make the pointed stars below.



6. Make the following concentric squares and equilateral triangles.

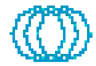


7. Make a net that will fold into a cube.

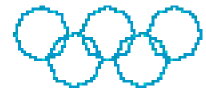


8. Make these figures:

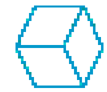
a.



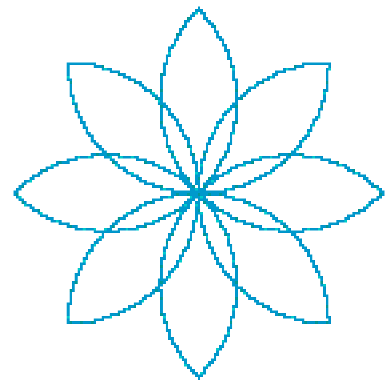
b.



c.



d. A flower



9. The following procedure will make a quadrilateral:

```
TO QUAD :M :N :A
REPEAT 2 [FD :M RT :A FD :N RT 180 - :A]
END
```

Modify the QUAD procedure so that the longest side of the quadrilateral will be parallel to the bottom of the paper.